Risk-adjusted Decision Making for Sustainable Development

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Notion “VUCA” was introduced by the US Army College to describe the world as resulting from the end of the Cold War.
VUCA-world challenges require VUCA-powerful methods to derive effective and efficient solutions

“How can it be that mathematics, being after all a product of human thought, which is independent of experience, is so admirably appropriate to the objects of reality?”

Albert Einstein, 1921
Optimization problem under chance constraints – general formulation

Minimize $f(x)$

$h_j(x) \geq 0 \quad j = 1, \ldots, k$

$\text{Prob}[g_i(x, \omega) \leq 0] \geq q_i \quad i = i, \ldots, m, \quad \omega \in \Omega$

$x$ is a vector of first-stage (strategic) decision variables

$f$ is an objective function

$h_i$ are functions representing deterministic constraints

$g_i$ are functions defining probabilistic chance constraints

$q_i$ are specified target reliabilities

$\omega$ is a vector of random variables that represent uncertain parameters

$\Omega$ is a set of all possible values of $\omega$
Application to a Water-Energy-Food nexus management problem
Application: Water-Energy-Food nexus

Minimize $f(x)$

$$h_j(x) \geq 0 \quad j = 1, \ldots, k$$

$$\text{Prob}[g_i(x, \omega) \leq 0] \geq q_i \quad i = i, \ldots, m, \quad \omega \in \Omega$$

- Focus: A region consisting of sub-regions, each producing coal and growing crops – both require water which is scarce. Demands for coal and crops are given.
- Objective function and constraints are linear functions of decision variables.
- Major decision variables are the amounts of production of coal by different technologies and the amounts of crops produced in each sub-region -- total \# of sub-regions $X$ number of crops $X$ number of coal technologies variables.
- Additional decision variables: Amounts of coal and crops transported across regions.
- Deterministic constraints describe food and energy security.
- Probabilistic constraints describe the availability of water.
- Uncertainty is the water supply.
Solution method – equivalent optimization problem with a penalty term

\[
\text{Minimize } \mathbb{E}[F(x, \omega)]
\]
\[
F(x, \omega) = f(x) - \sum_{i=1}^{m} \alpha_i y_i(x, \omega)
\]
\[
h_j(x) \geq 0 \quad j = 1, \ldots, k
\]
\[
y_i(x, \omega) = \min\{0, -g_i(x, \omega)\}
\]

- If the water constraint is satisfied \((g_i < 0)\), the penalty term is zero; if it is not satisfied, the unsatisfied water requirement is penalized
- **Target probabilities** \(q_i\) translate into penalty coefficients \(\alpha_i\)!!!
Interpretation: two stages

• First stage – “strategic” decisions – taken and implemented before the uncertainty is realized – e.g., installment of water saving technologies

\[
\text{Minimize } E[F(x, \omega)] \\
F(x, \omega) = f(x) - \sum_{i=1}^{m} \alpha_i y_i(x, \omega) \\
h_j(x) \geq 0 \quad j = 1, ..., k
\]

• Second stage – “adaptive” decisions – taken and implemented after the uncertainty is realized – e.g., “importation” of water from outside the region or reduction of the household consumption

\[
y_i(x, \omega) = \min\{0, -g_i(x, \omega)\}
\]
Practical approach to compute the expected value

\[ F(x, \omega) \approx f(x) - \sum_{i=1}^{m} \alpha_i \sum_{s=1}^{S} p_s \min\{0, -g_i(x, \omega_s)\} \]

• The expected value of the penalty term is replaced by the sample mean
• Sample is based on observations in the past and/or predictions for future

Source: Ermoliev and Wets (1988)
Model outline

Source (here and in the next slides): Gao et al (2021)
Case study area: Shanxi Province, China

Water availability in Shanxi varies across prefectures and time.
Reliability levels under different penalties, and comparison of the stochastic and deterministic solutions.
Solution costs

Total, first-stage, and the expected value of second-stage costs of the stochastic solution at different penalty levels. The numbers 742, 962, and 1412 represent optimal volumes of water storage, in million m$^3$. Percentages in parenthesis indicate the reliability levels that can be achieved at the corresponding penalty level due to the deployment of the indicated technologies in the first stage.
Amount of water storage at the second stage and water withdrawal by crops throughout the entire province at different penalty levels. At penalty levels lower than 12 RMB per ton, water storage is zero. Percentages in parenthesis indicate the reliability levels that can be achieved at the corresponding penalty level by establishing water storage at the indicated capacity in the first stage.
Optimal allocation of production

Optimal crop production in each sub-region under 11 RMB per ton of water penalty (left part of the figure) and 12 RMB per ton of water penalty (right part of the figure).
What is the benefit of information?

\[
VSS = EF(x^{E\omega}, \omega) - EF(x^*, \omega)
\]

\[
EVPI = EF(x^*, \omega) - Ef(x^\omega, \omega)
\]

\(x^*\) is the first-stage solution in the two-stage stochastic optimization problem

\(x^\omega\) is the (first-stage) solution in the deterministic optimization problem

\[
\text{Minimize } f(x) - \sum_{i=1}^{m} \alpha_i \min\{0, -g_i(x, \omega)\}
\]

\[
h_j(x) \geq 0 \quad j = 1, ..., k
\]

The expected value of perfect information (EVPI) and the value of the stochastic solution (VSS) relative to the total cost of the stochastic solution under different penalties.
Robustness of the stochastic solution

For each water availability scenario, the total costs of the deterministic and stochastic solutions for a penalty of 30 RMB per ton of water.
Application to Pollution Control in Surface Waters
Pollution control: Eutrophication in surface waters

Lake Erie, North America

Baltic Sea
Source: The Guardian, 2020
Fertilizers and pollution control

- Land allocation
- Fertilizer application
- Cover crops adoption path

Environmental regulations

Phosphorus in soil

Uncertainty

Phosphorus in ground water

Chance constraint problem

- Dynamics of the Phosphorus cycle taken into account
- Expected profit is maximized over Phosphorus application, crop allocation and cover crops
- Uncertain emissions into the lake need to be limited with high reliability

\[
\max_{\delta_i, \theta_i, F_i} \sum_{t=1}^{\infty} \beta^t \sum_i \delta_i(t) \mathbb{E} \left[ \pi_i \left( \theta_i(t), F_i(t), \omega \right) \right]
\]

s.t. state equations

\[
\mathbb{P} \left[ E^{sa}(t, \omega) \leq \eta^{sa} \right] \geq 1 - \epsilon^{sa}
\]

\[
\mathbb{P} \left[ E^{ss}(t, \omega) \leq \eta^{ss} \right] \geq 1 - \epsilon^{ss}
\]
Reformulation as two-stage problem with recourse

Fertilizer application, crop allocation, and farming practices (strategic decisions) need to be chosen wisely to avoid paying too large penalties (adaptive decisions)

\[
\begin{align*}
\max_{\delta_i, \theta_i, F_i} & \quad \sum_{t=1}^{\infty} \beta^t \left( \sum_i \delta_i(t) \mathbb{E} \left[ \pi_i \left( \theta_i(t), F_i(t), \omega \right) \right] \right) \\
\text{s.t.} & \quad \text{state equations}
\end{align*}
\]
Relationship between risk level and corresponding cost
Robust solution provides significant tighter guidelines for fertilizer application

- Fertilizer application based on mean emission rates has historically led to harmful algal blooms
- Robust fertilizer application rate 16% lower than application based on mean emission rate
Application to Financing Mechanisms for Sustainable Food Security
Risk management for sustainable food security

- Population is growing very quickly in West-Africa
- Farmers in the region are faced with extreme weather events
- Stable incomes are necessary to avoid poverty traps and allow for investment in efficient farming technologies.
- Can local food production keep up with population growth?
- Is it possible to provide more stable incomes to local farmers by means of a catastrophe fund?
- How much risk pooling needs to occur to ensure food security and/or solvency of the catastrophe fund?
Modeling food security and financial risk transfer (i)

- Establish dependence structure of crop yields in West-Africa

- Define crop yield projections for the coming 25 years
  - Uncertainty: random vector of yields over different clusters
  - Non-stationary process: expected yield is varying with time
Modeling food security and financial risk transfer (ii)

- **SO model** that
  - Minimizes cultivation cost
  - Food security needs to be guaranteed with given reliability level
  - Catastrophe fund needs to stay solvent after catastrophic event
  - Crop choice and land allocation (strategic decisions) need to be chosen wisely to avoid importing food at higher price and taking loan (adaptive decisions)

\[
\begin{align*}
\min_{x(t)} \mathbb{E} & \left[ \sum_{t=t_0}^{T_{\text{fin}}} \sum_k \sum_j C_j \cdot x_{j,k,t} \right] \\
\text{s.t.} & \mathbb{P} \left[ \sum_k \sum_j Y_{j,k,t} \cdot x_{j,k,t} \cdot a_j \geq A_t \right] \geq \alpha_F \\
& \mathbb{P} \left[ G_{\text{fin}}(x, Y) \geq 0 \right] \geq \alpha_S
\end{align*}
\]

- Minimize expected cultivation costs incurred over time, clusters, and crops
- Food security constraint depending on random yield and land allocation
- Solvency constraint for the catastrophe fund after a catastrophic event
Managing food security and insolvency risk

- We consider different population scenarios and analyze the effects on allocated land.
- Food security probability at 95% and solvency probability at 85%.
- Food security constraint and solvency constraint not active at the same time.
- Maize substituted by rice once limit of arable land is reached.
- Currently, arable land is sufficient to meet food demand with high probability, but population growth is a critical factor for sustainable food security in West Africa.

![Fixed yield distribution and yield projections included](image)
Government interventions to finance catastrophe fund

- We consider three policy levers that can be used by governments to manage food security and stabilize incomes.
- Tax rate affects time needed to build up the fund.
- Share of guaranteed income requires large initial overproduction.
- Increased risk level reduces the expected losses to be refunded to farmers.
Spatial correlation and risk pooling

- We quantify the effect of risk pooling by evaluating the total amount of food imports and the total debt that needs to be sustained for different levels of cooperation.
- Collaboration between different clusters improves the feasibility to achieve food security and solvency of the financing mechanism.
- Limited levels of cooperation result in large benefits for the food security, and cooperation over larger areas is necessary to improve the solvency objective.

Developed depending on collaboration of clusters

- Food security probability 99%
- Solvency probability 95%
- Covered risk level 5%
- Tax rate 1%
- Percentage of expected income guaranteed 90%